

Orthogonal Convolutional Neural Networks

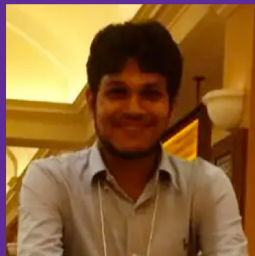
Jiayun Wang



Yubei Chen



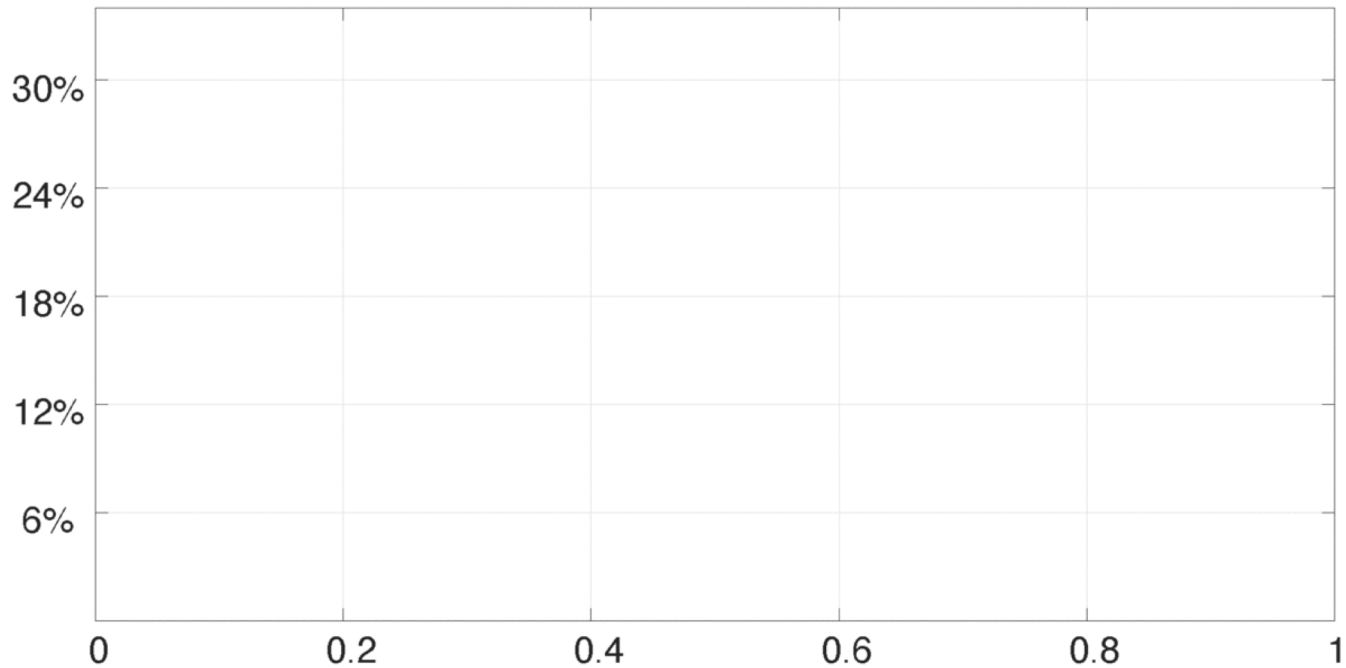
Rudrasis Chakraborty



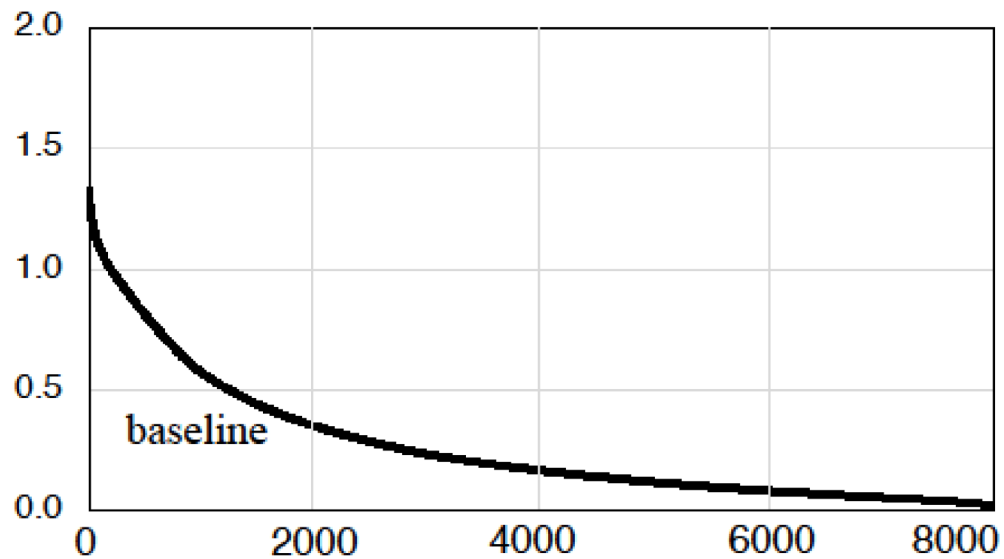
Stella X. Yu



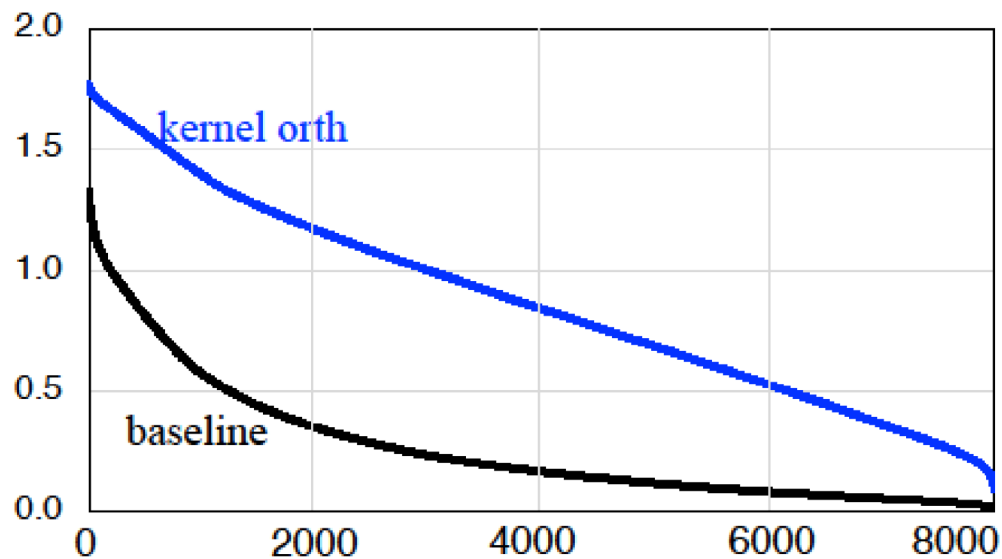
Filter similarity increases with depth



A typical conv layer has highly irregular spectrum



Kernel orthogonality is widely used as a regularization



Saxe et al. 2014

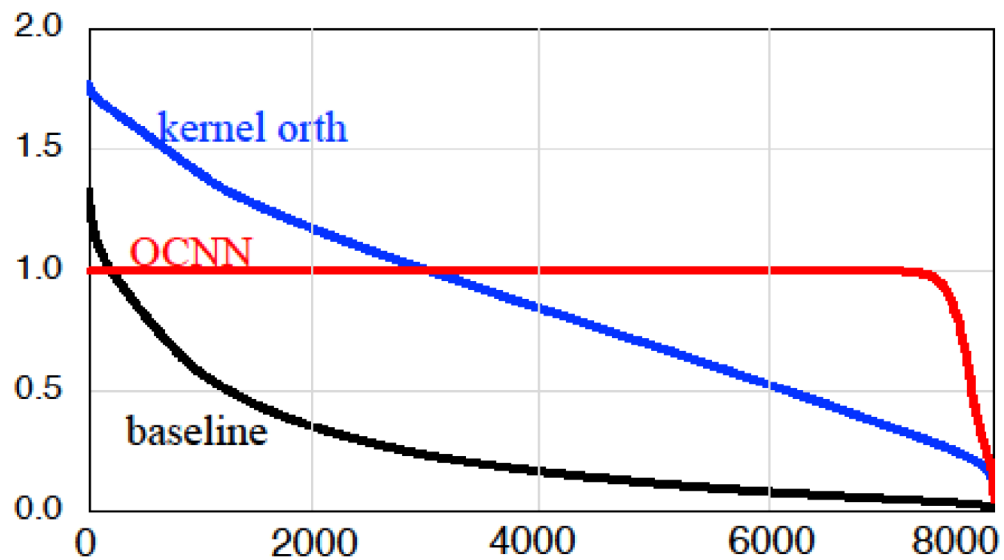
Dorobantu et al. 2016

Rodriguez et al. 2017

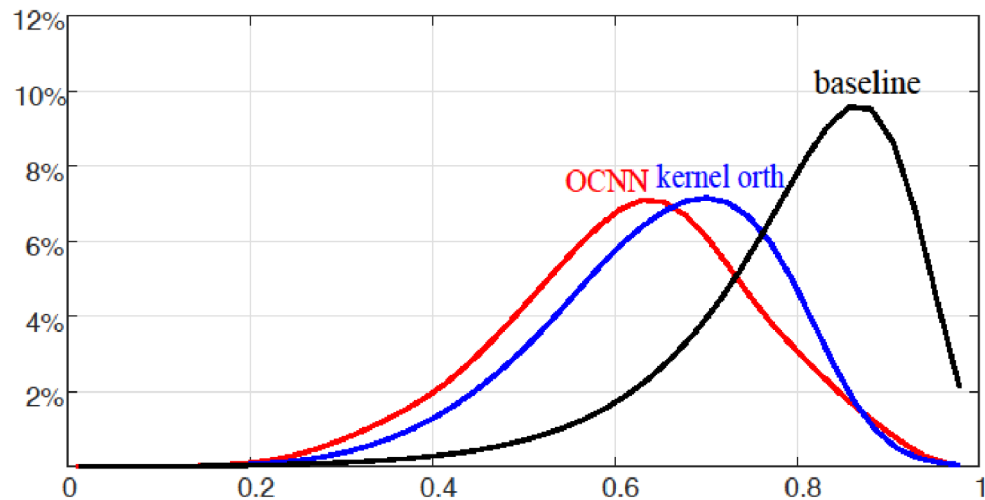
Bansal et al. 2018

...

OCNN can do even better

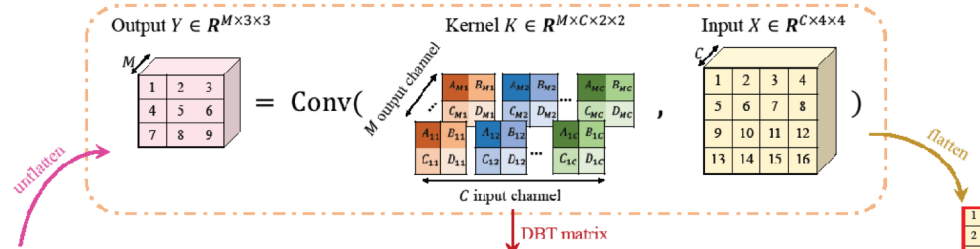


Filter diversity improvement with OCNN



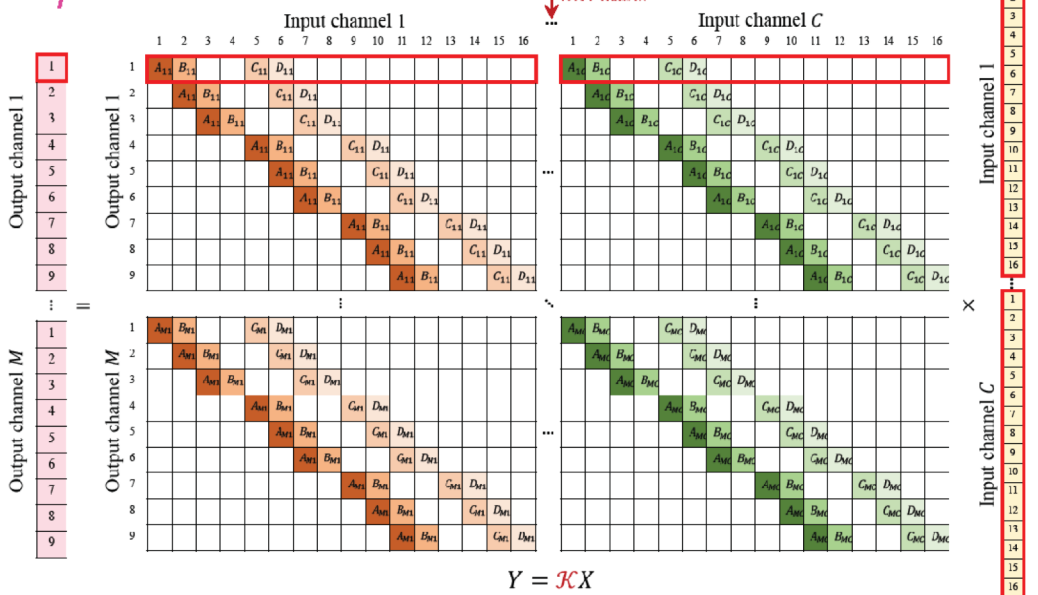
Convolution is an efficient matrix-vector multiplication

$$Y = \text{Conv}(K, X), \text{ stride } 1$$



Convolution:

$$Y = K * X$$



Matrix-vector form:

$$K \rightarrow \mathcal{K}, Y = \mathcal{K}X$$

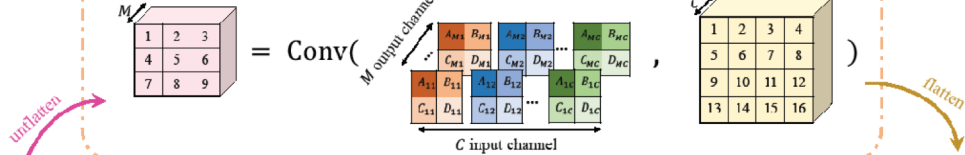
Convolution is an efficient matrix-vector multiplication

$$Y = \text{Conv}(K, X), \text{ stride } 1$$

Output $Y \in \mathbb{R}^{M \times 3 \times 3}$

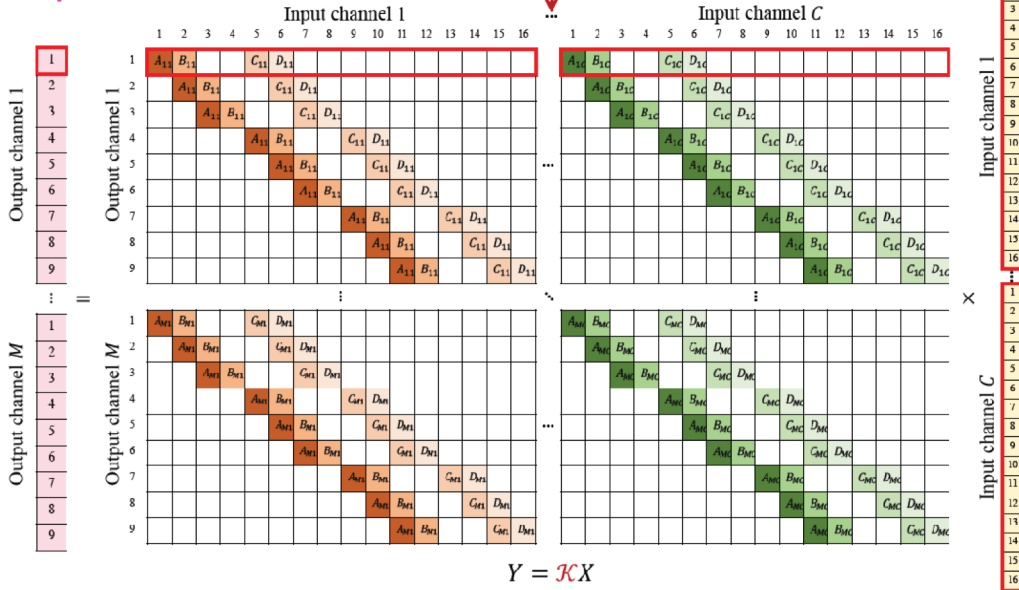
Kernel $K \in \mathbb{R}^{M \times C \times 2 \times 2}$

Input $X \in \mathbb{R}^{C \times 4 \times 4}$



Convolution:

$$Y = K * X$$

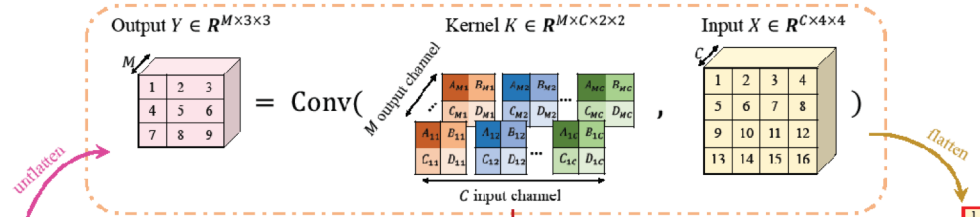


Matrix-vector form:

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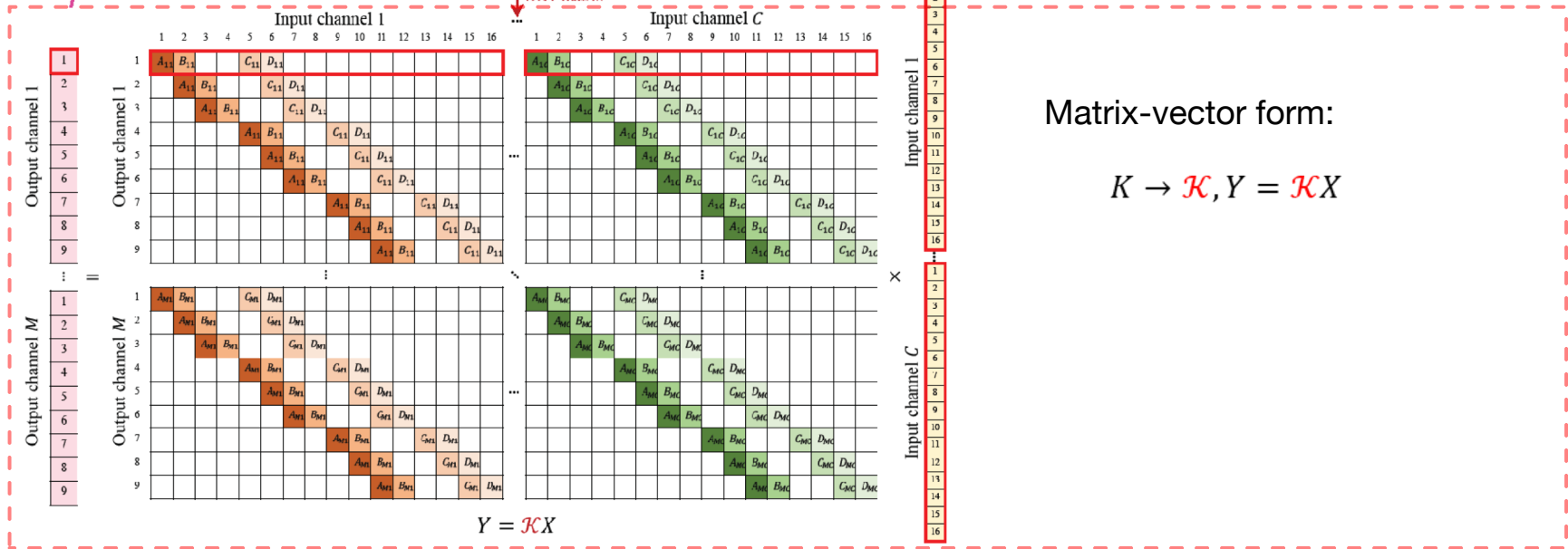
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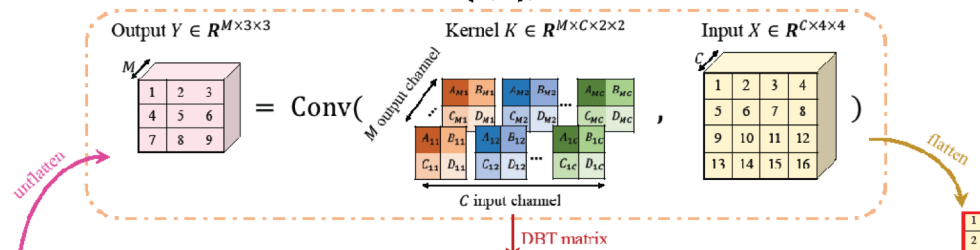


Matrix-vector form:

$$K \rightarrow \mathcal{K}, Y = \mathcal{K}X$$

Orthogonal convolution or orthogonal kernel?

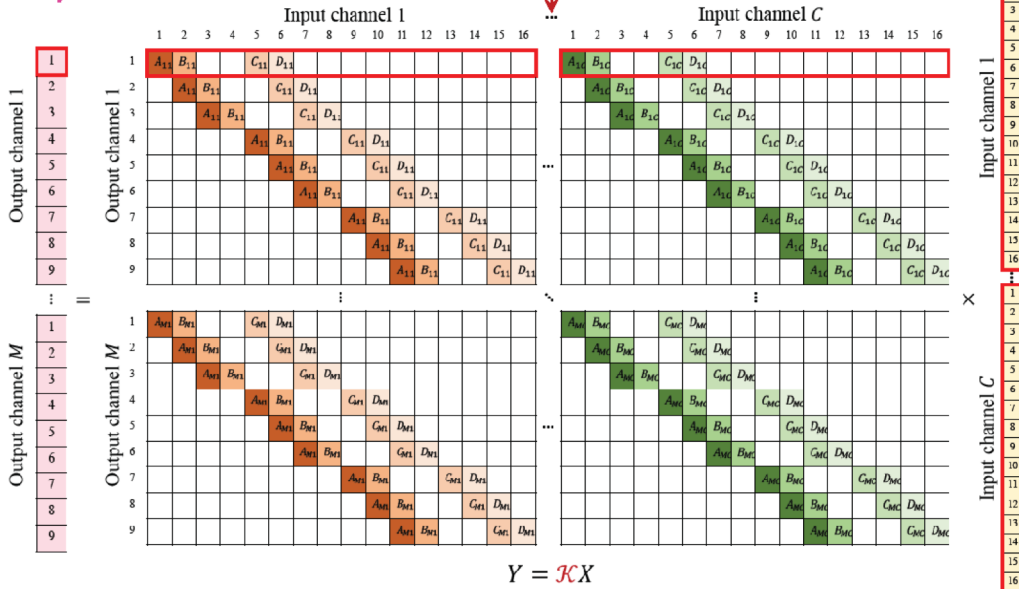
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Convolution:

$$Y = K * X$$

kernel orthogonality: $KK^T = I$



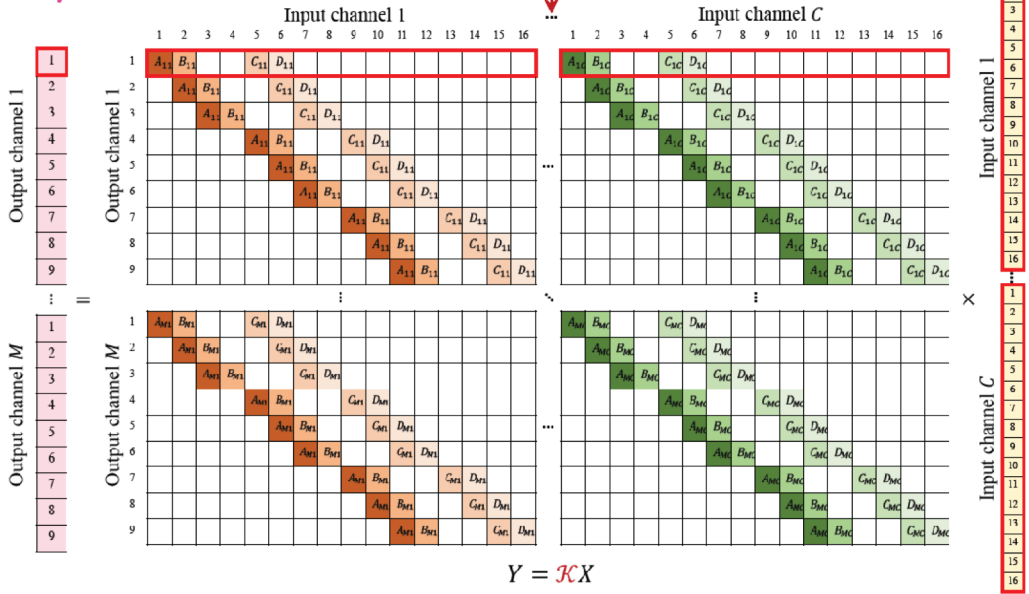
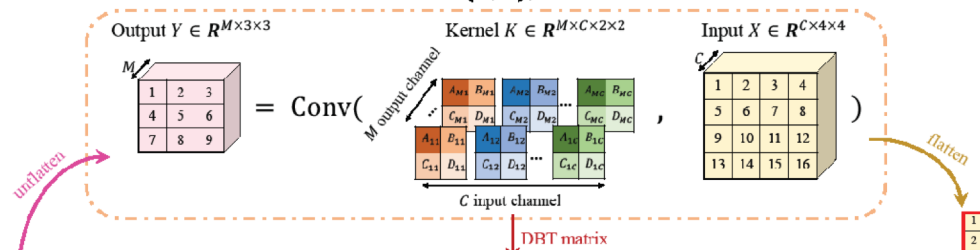
Matrix-vector form:

$$K \rightarrow \mathcal{K} Y = \mathcal{K}X$$

conv orthogonality: $\mathcal{K}\mathcal{K}^T = I$

Orthogonal convolution or orthogonal kernel?

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Convolution:

$$Y = K * X$$

kernel orthogonality: $KK^T = I$

Matrix-vector form:

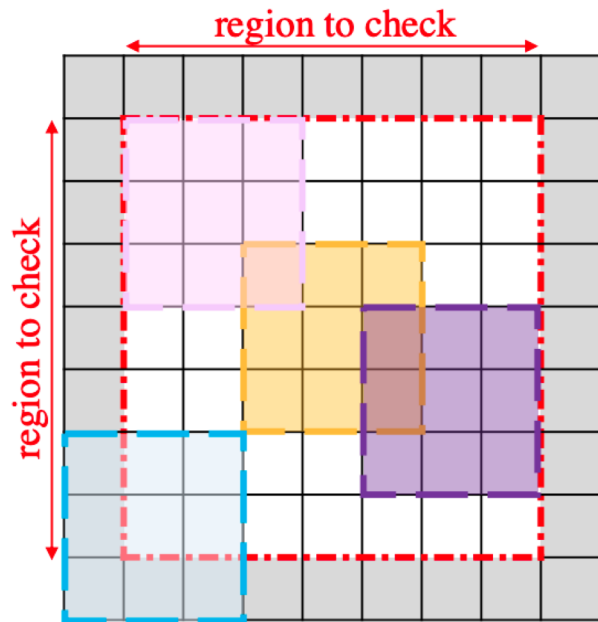
$$K \rightarrow \mathcal{K} Y = \mathcal{K}X$$

conv orthogonality: $\mathcal{K}\mathcal{K}^T = I$

$$\mathcal{K}\mathcal{K}^T = I \Rightarrow KK^T = I$$

$$\mathcal{K}\mathcal{K}^T = I \not\Leftarrow KK^T = I$$

A fast algorithm for orthogonal convolution



- Kernel Orthogonality:

$$\begin{cases} \text{Conv}(K, K, \text{padding} = 0) = I_{r0} \\ \text{Conv}(K^T, K^T, \text{padding} = 0) = I_{c0} \end{cases}$$

- Convolutional Orthogonality:

$$\text{Conv}(K, K, \text{padding} = P, \text{stride} = S) = I_{r0}$$

Same # parameters and test time, only 9% more training time

Universal improvements

	Task	Metric	Gain
Image Classification	CIFAR100	classification accuracy	3%
	ImageNet	classification accuracy	1%
	semi-supervised learning	classification accuracy	3%
Feature Quality	fine-grained image retrieval	kNN classification accuracy	3%
	unsupervised image inpainting	PSNR	4.3
	image generation	FID	1.3
	deep metric learning	NMI	1.2
Robustness	black box attack	attack time	7x less



Thanks

