Neural Operators Accelerate 3D Photoacoustic Computed Tomography

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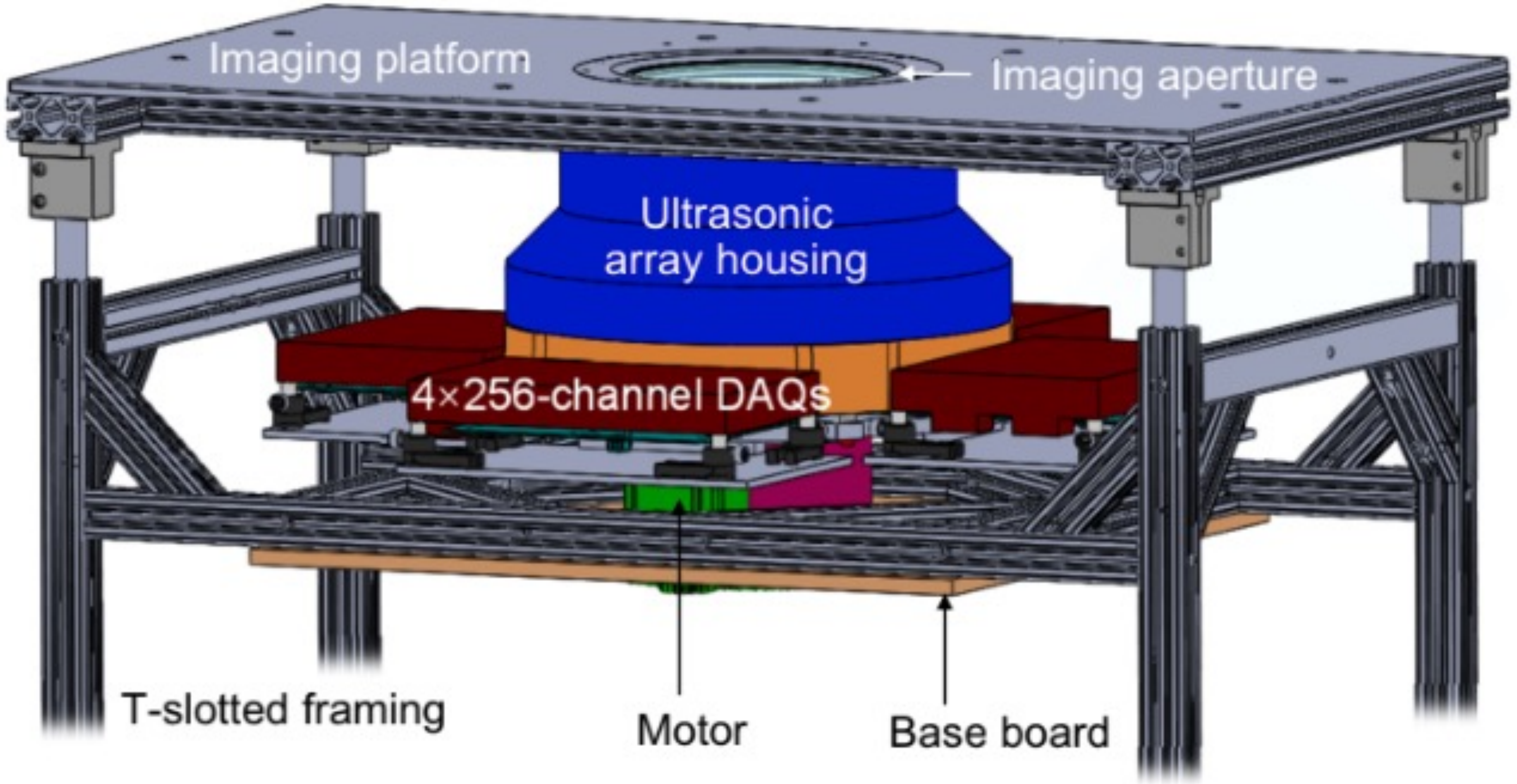
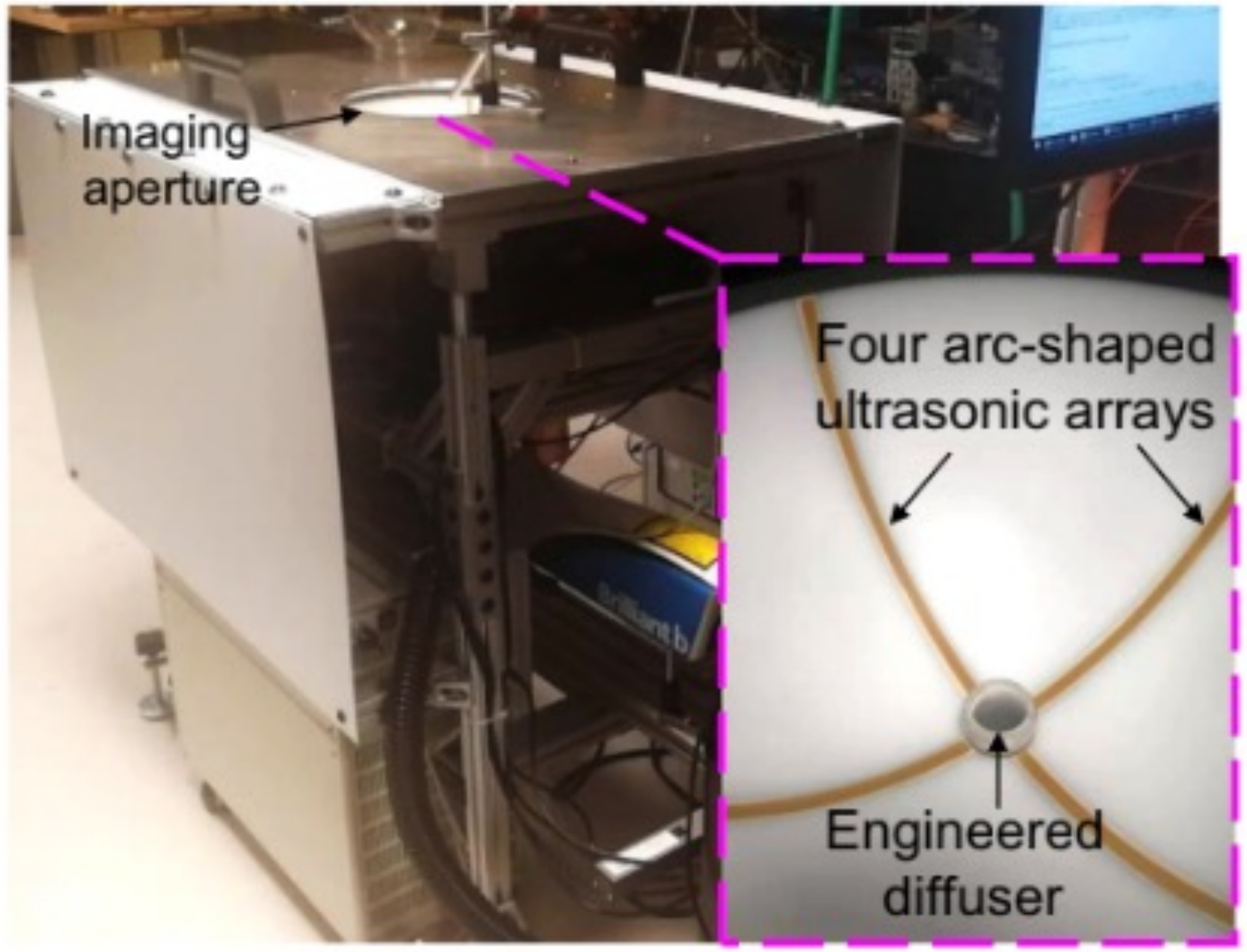
Hardware: 1k System (w/ rotated arcs)

Setup

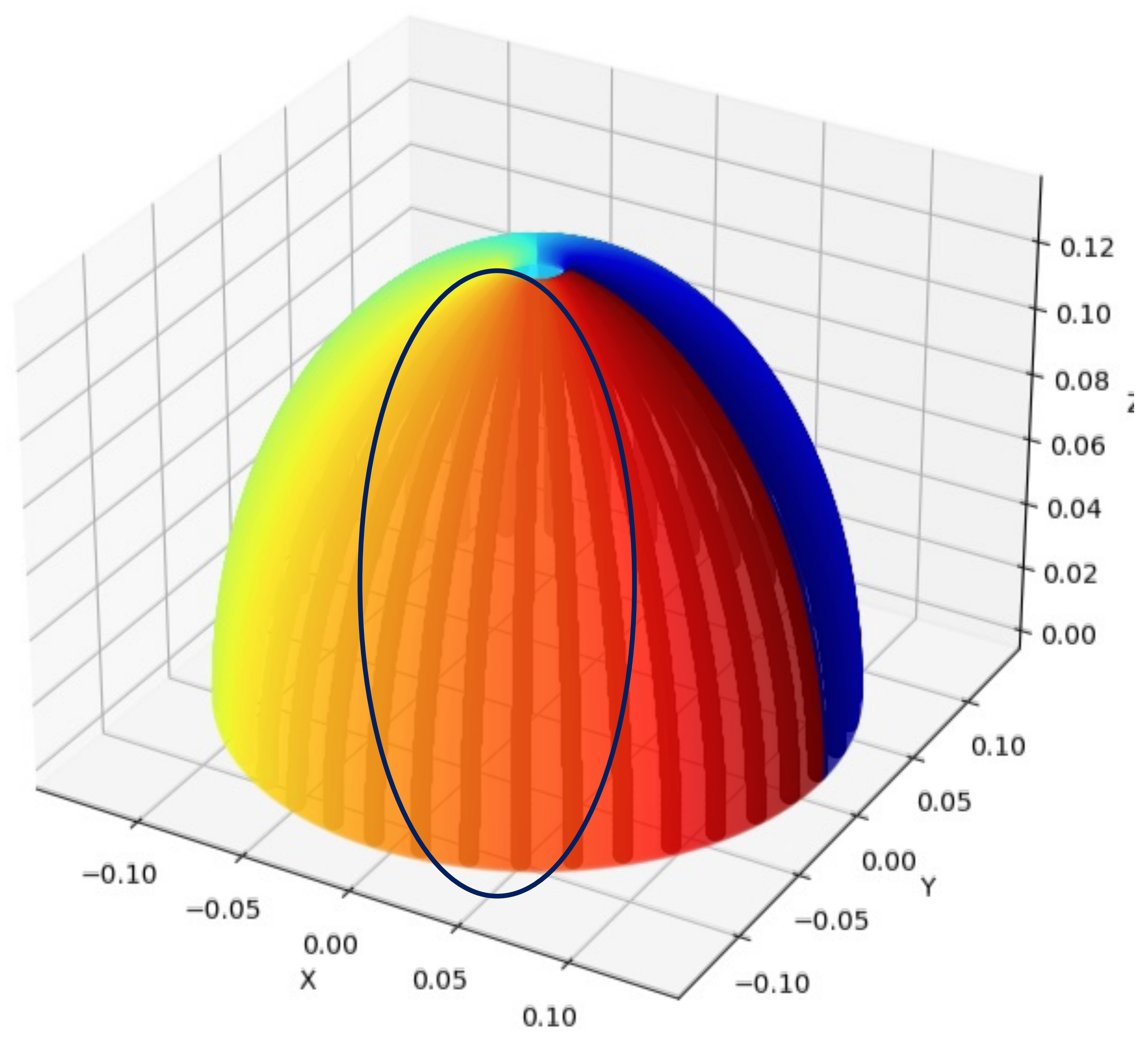




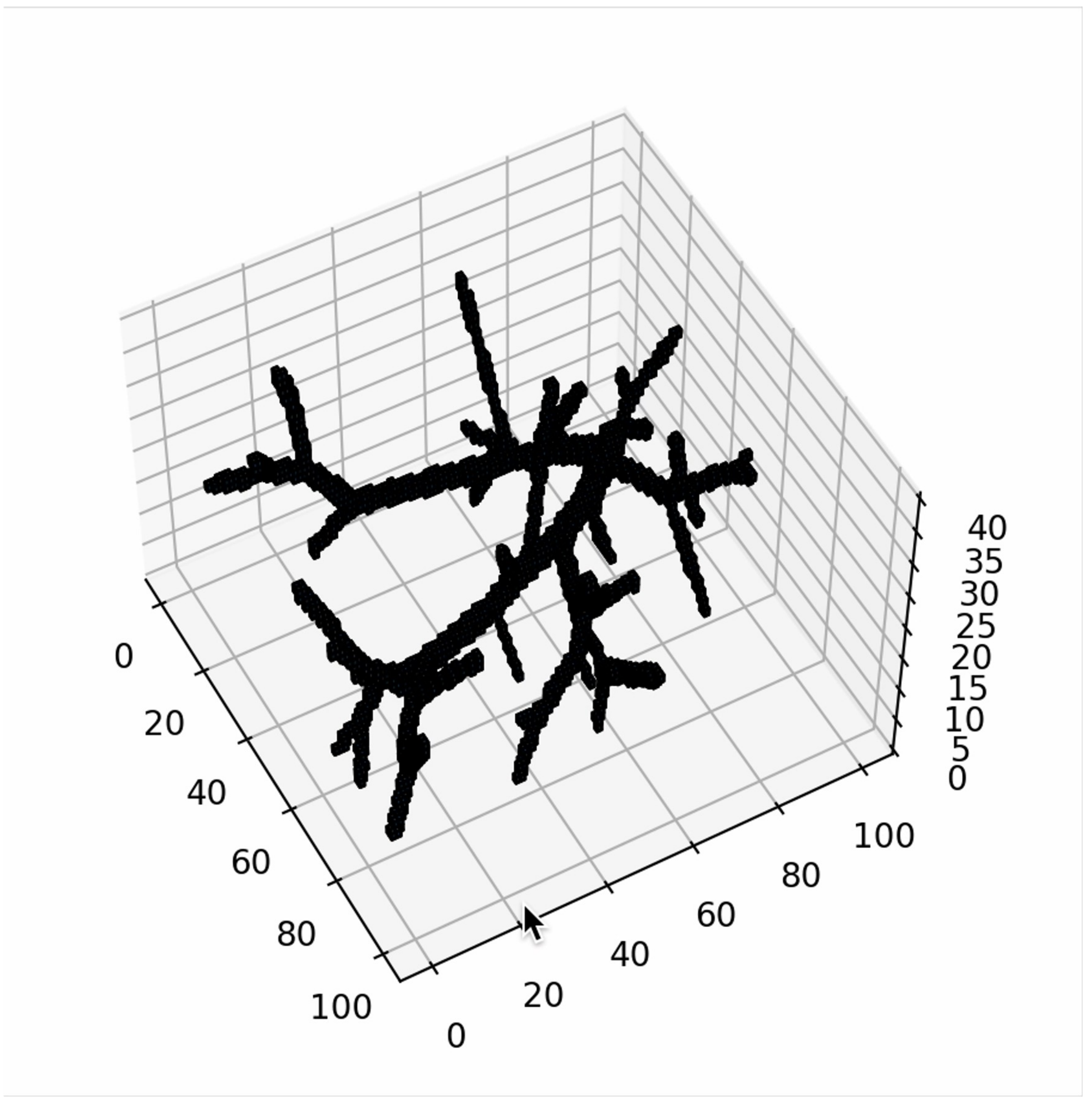
High-speed 3D PACT (2021)

Motivation: Compressed Sensing PACT

- Consistent reconstruction from **undersample measurement**
  - Reduce scan time
  - Low-cost PACT system (with fewer transducers)
  - Limited angle



Measurement Ψ



Source P

The Forward and Inverse Problem

- The imaging process be considered as solving the following Helmholtz equation

wave number

$$\nabla^2\Psi(x) + k^2\Psi(x) = \mathbf{j}wP(x)$$

wave function　　　　source

- **Forward**: Source to observed RF

$$P(x) \rightarrow \Psi(x)$$

Numerically, the forward model is $\Psi(x) = AP(x)$, A is the forward operator

- **Inverse**: Observed RF to reconstruct source

$$\Psi(x) \rightarrow \hat{P}(x)$$

Goal: learn an inverse operator with ML
which reconstructs high-quality image $P(x)$

Inverse operator A* is computation-expensive

Conventional Solver and ML

- Back projection solver (1 step)

$$\hat{P} = A^{*}\Psi$$

Conventional Solver and ML

- Back projection solver (1 step)

$$\hat{P} = A^*\Psi$$

- Iterative solver (5-10 steps)

$$\hat{P} = \arg\min_{P \geq 0} \|AP - \Psi\|^2 + \mathcal{R}(P),$$

where $A : \mathbb{R}^N \to \mathbb{C}^M$, $P \in \mathbb{R}^N$, and $\Psi \in \mathbb{C}^M$, with $N = 200 \times 200 \times 160$ and M being the number of transducers multiplied by the number of frequency modes (149). $\mathcal{R}(P)$ is a regularizer like TV regularization.

A can be as large as $15\text{k}\times 6\text{k}$ (10s scan)

- ML learns the inverse operator to reconstruct source with parameter Θ

$$\hat{P} = f_\Theta(\Psi)$$

Methods – Overview

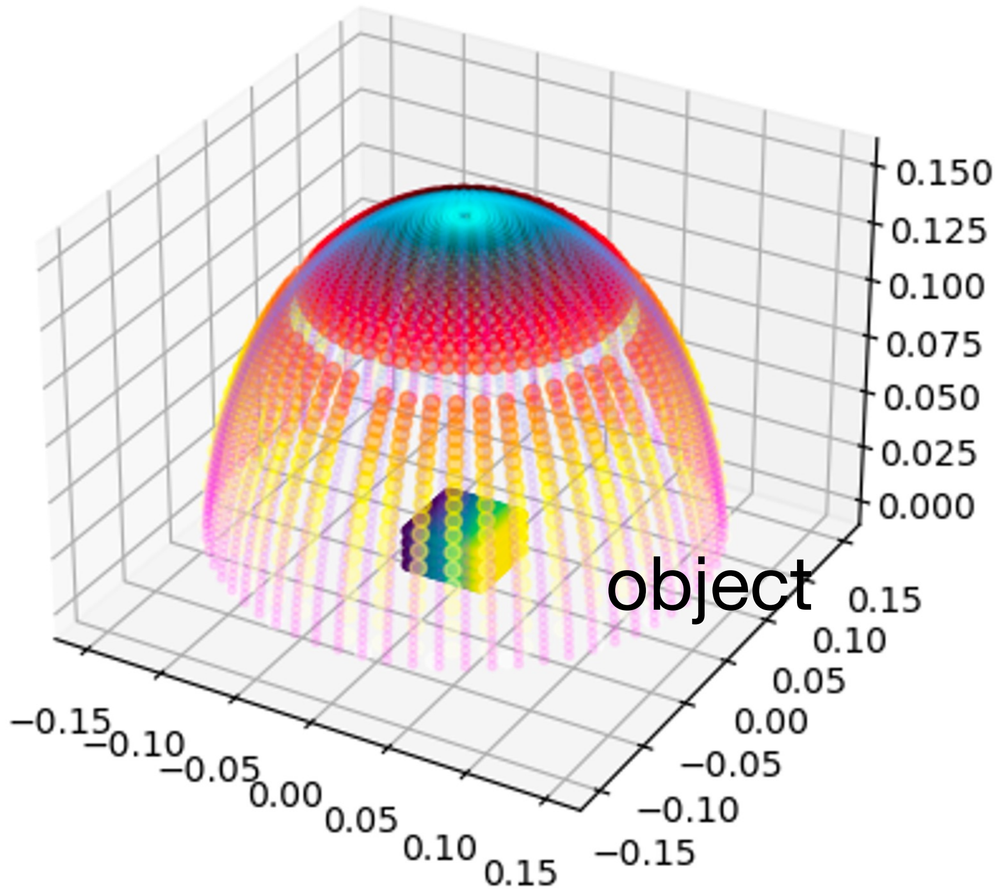
Helmholtz Equation

wave number

$$\nabla^2\Psi(x) + k^2\Psi(x) = jwP(x)$$

wave function

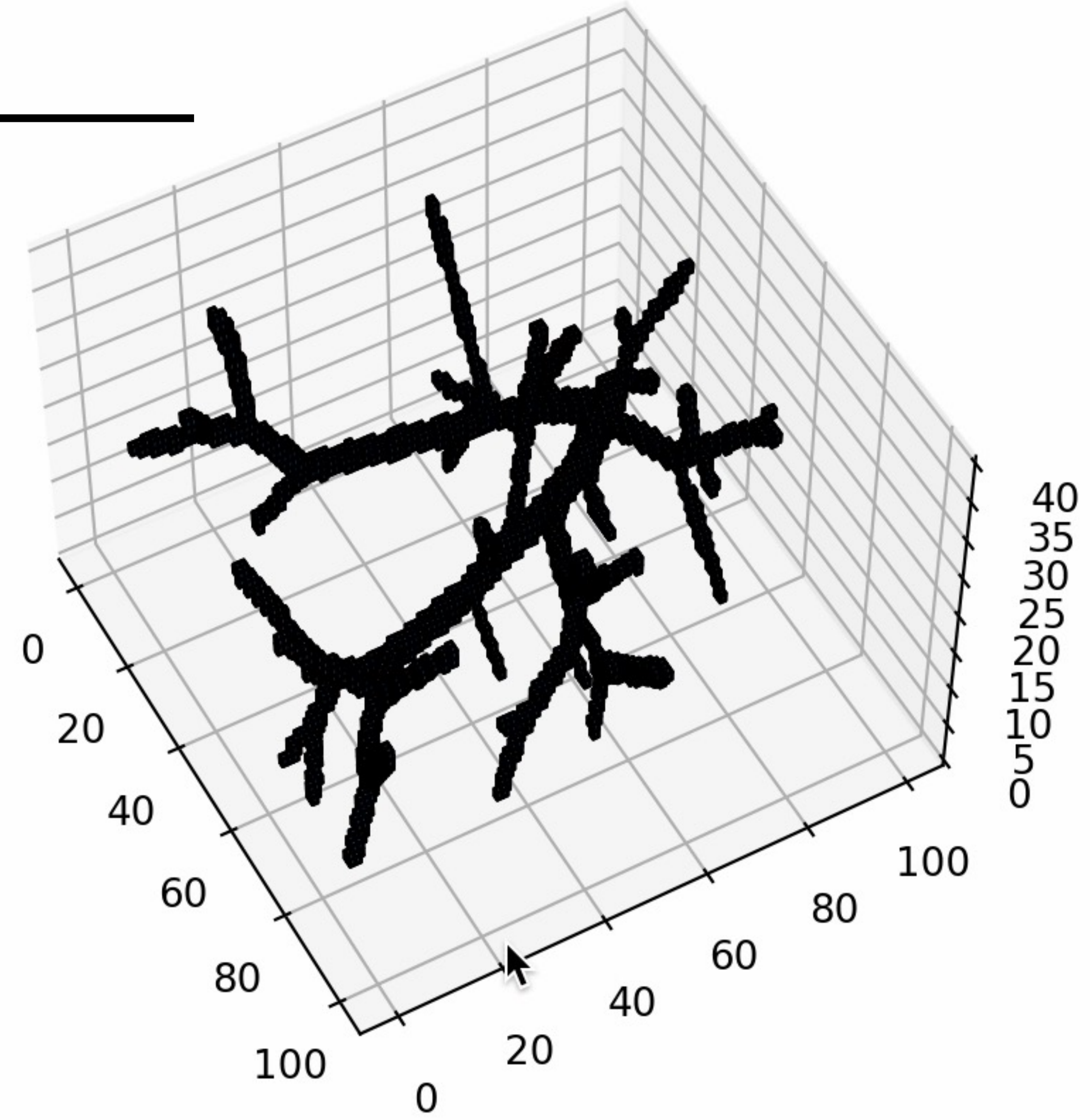
source



PDE loss
(physics-informed)

Transducer index × Time

Resolution-invariant
Neural operator



3D Image –
200x200x160

Methods – Geometry-Informed Network

- RF signal is 3D (θ, ϕ, t)
- We use spherical coordinates to simplify computation
- The neural operator framework is resolution-agnostic

The framework learns in the function space and is resolution-invariant

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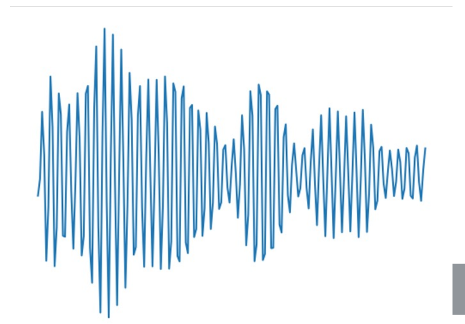
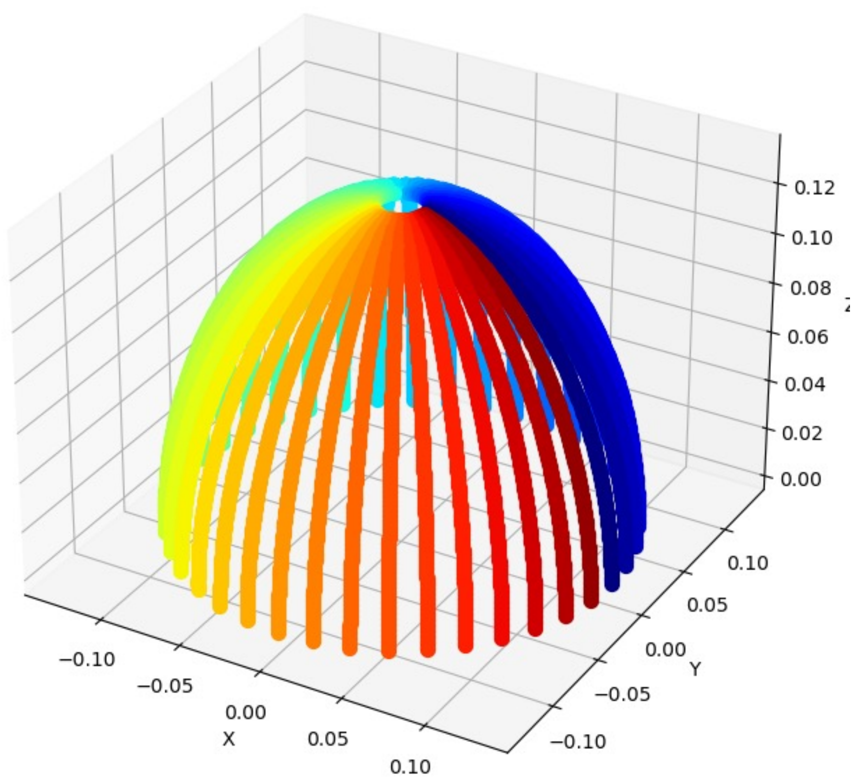
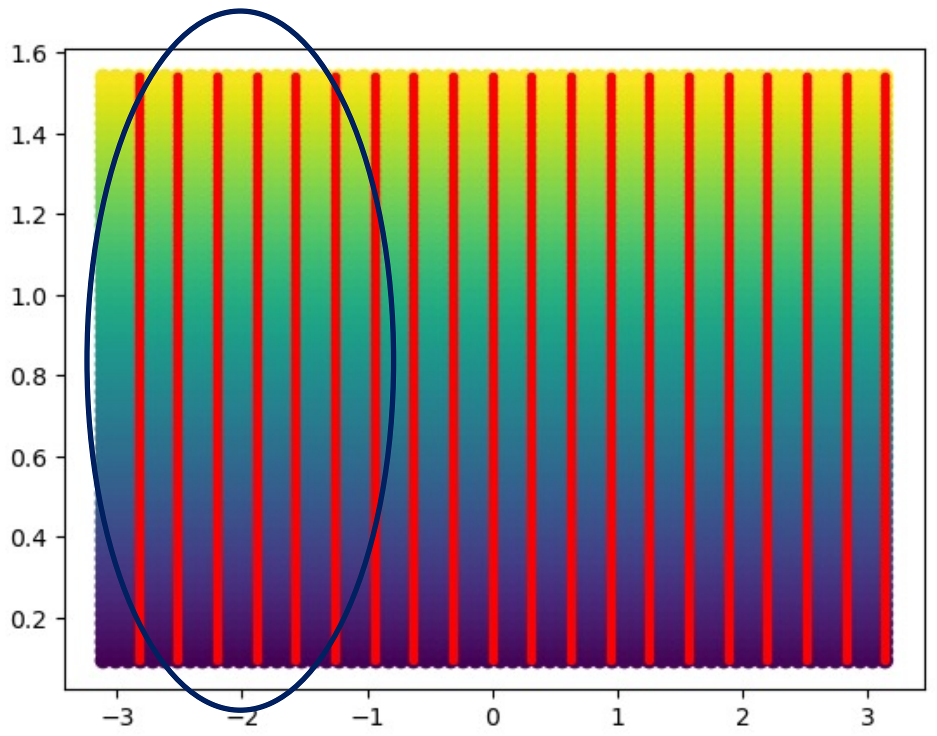


RF data



S^2 sphere in 3D

10s scan = 400 vertical lines
0.5s scan = 20 lines (visualization)



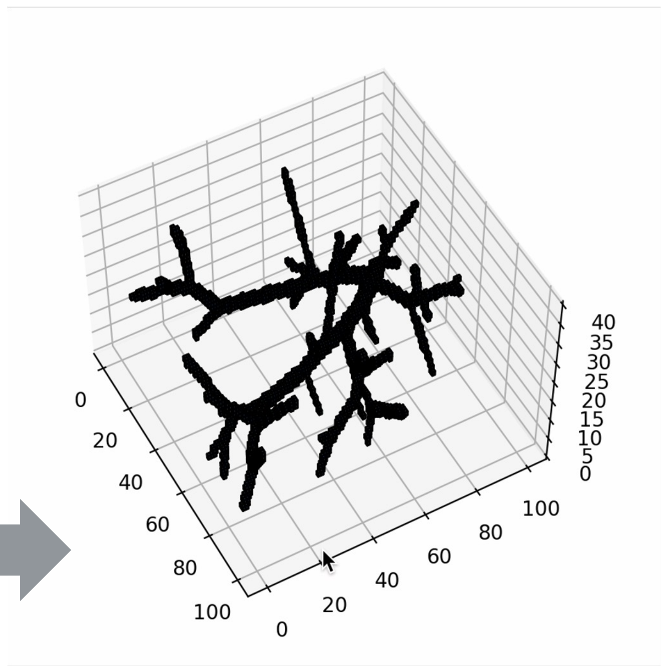
2D spherical coordinates

DISCO → FNO → UDNO

DISCO: resolution-agnostic convolution

FNO: global feature

UDNO: local feature



3D image (200x200x160)

The framework learns in the function space and is resolution-invariant

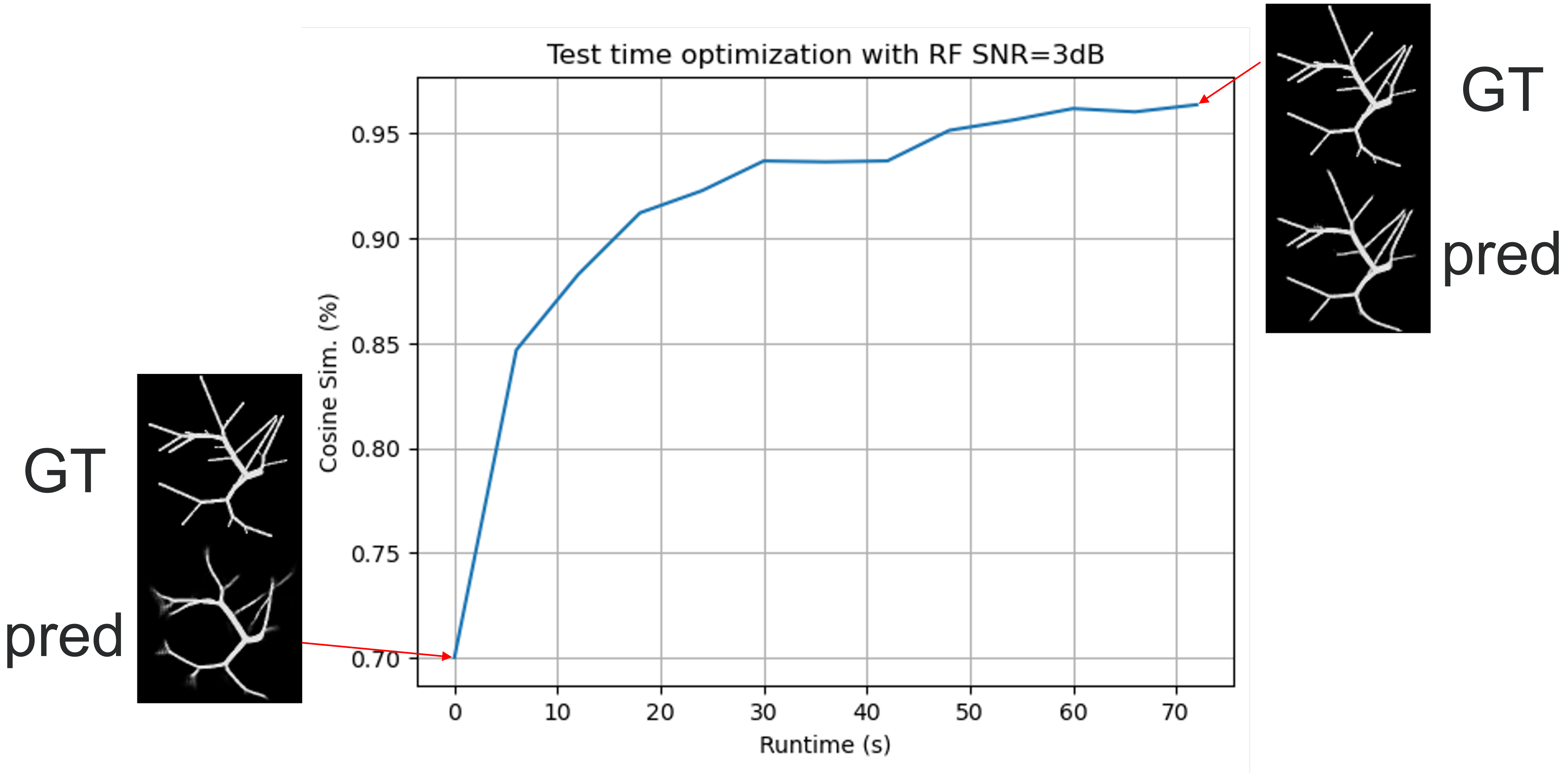
Methods - Test-Time Optimization

Solver Optimized on Neural Representations

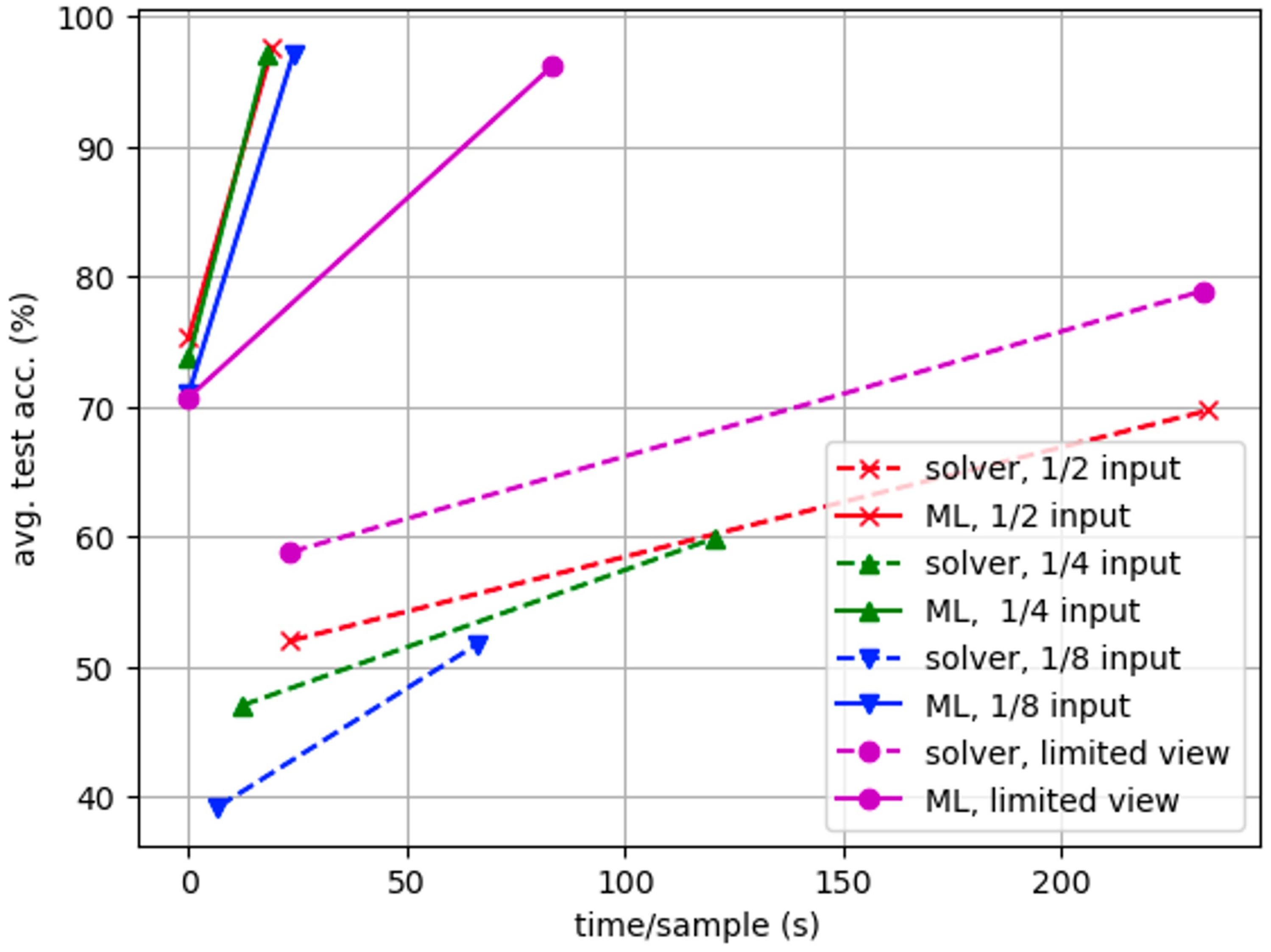
- After training the ML model, we can use test-time optimization to further improve reconstruction performance for a sample

$$\min \left\| A\hat{P}(x) - \Psi(x) \right\|_2 = \min_{\theta} \| A f_\theta(\Psi(x)) - \Psi(x) \|_2$$

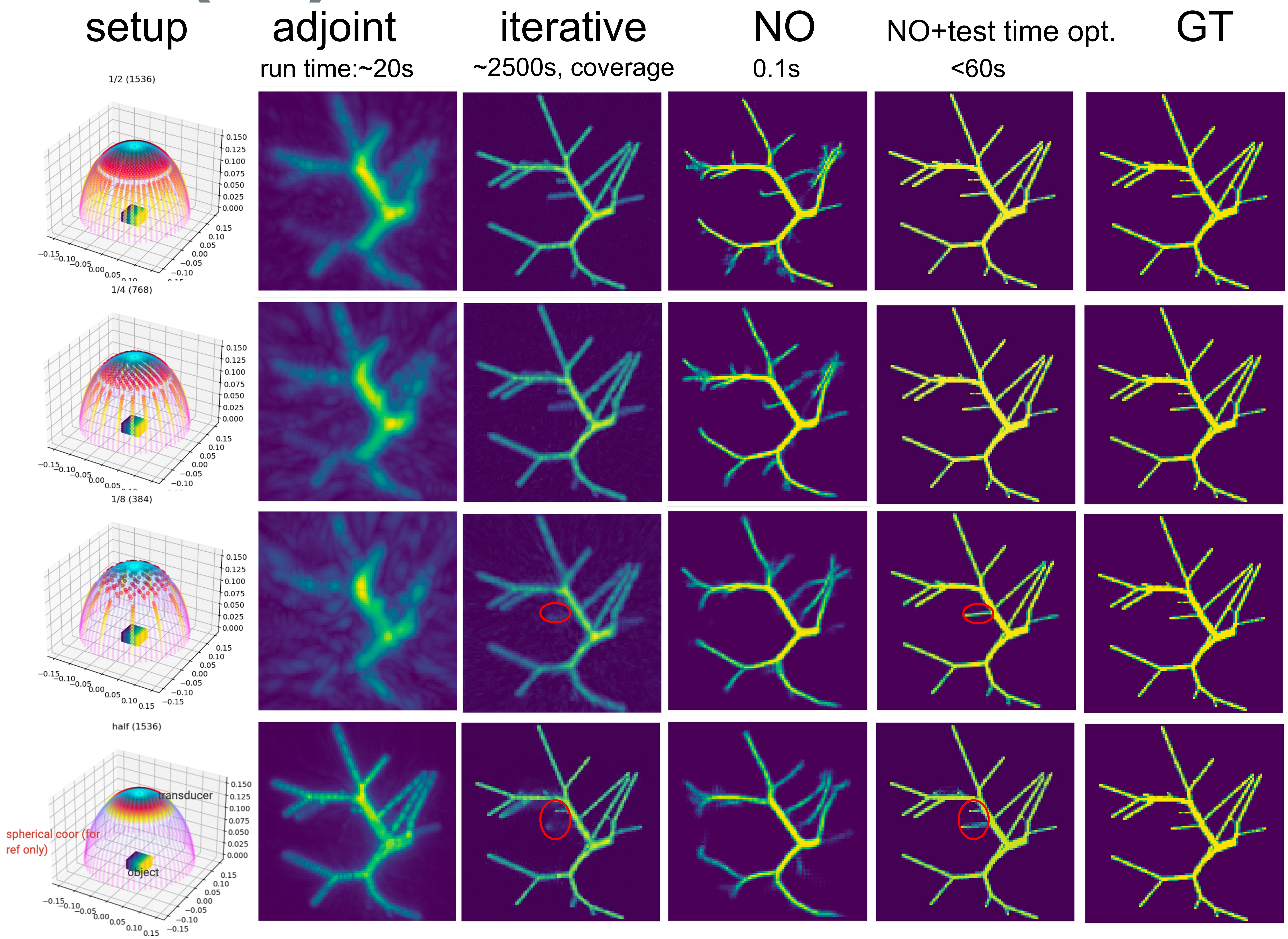
- Compared to PDE loss: PDE loss is optimized for all training images



Simulation Results (3k)



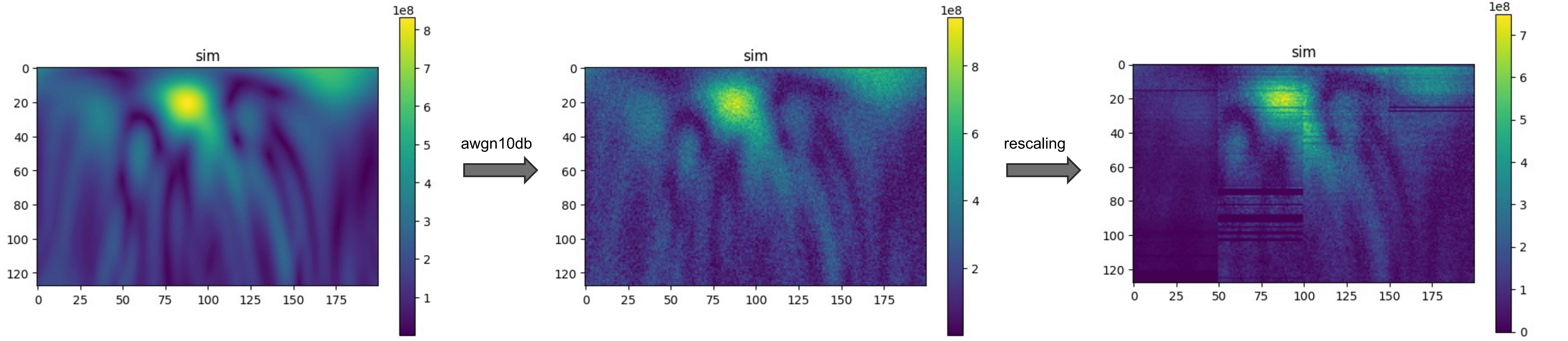
metric: cosine similarity
visualization: maximum projection on z-plane

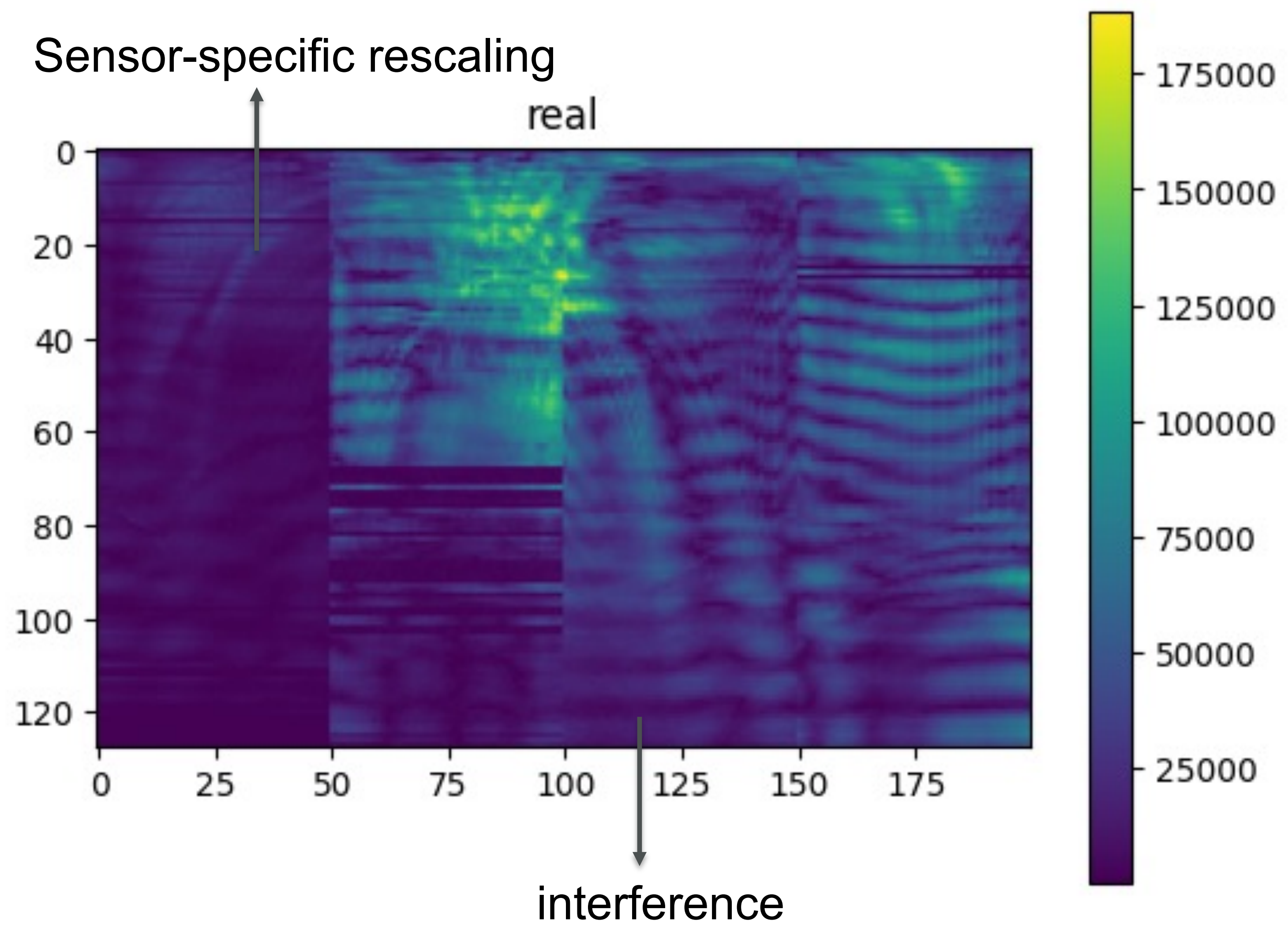


1k results pending

Reducing Sim and Real Gap (1k)

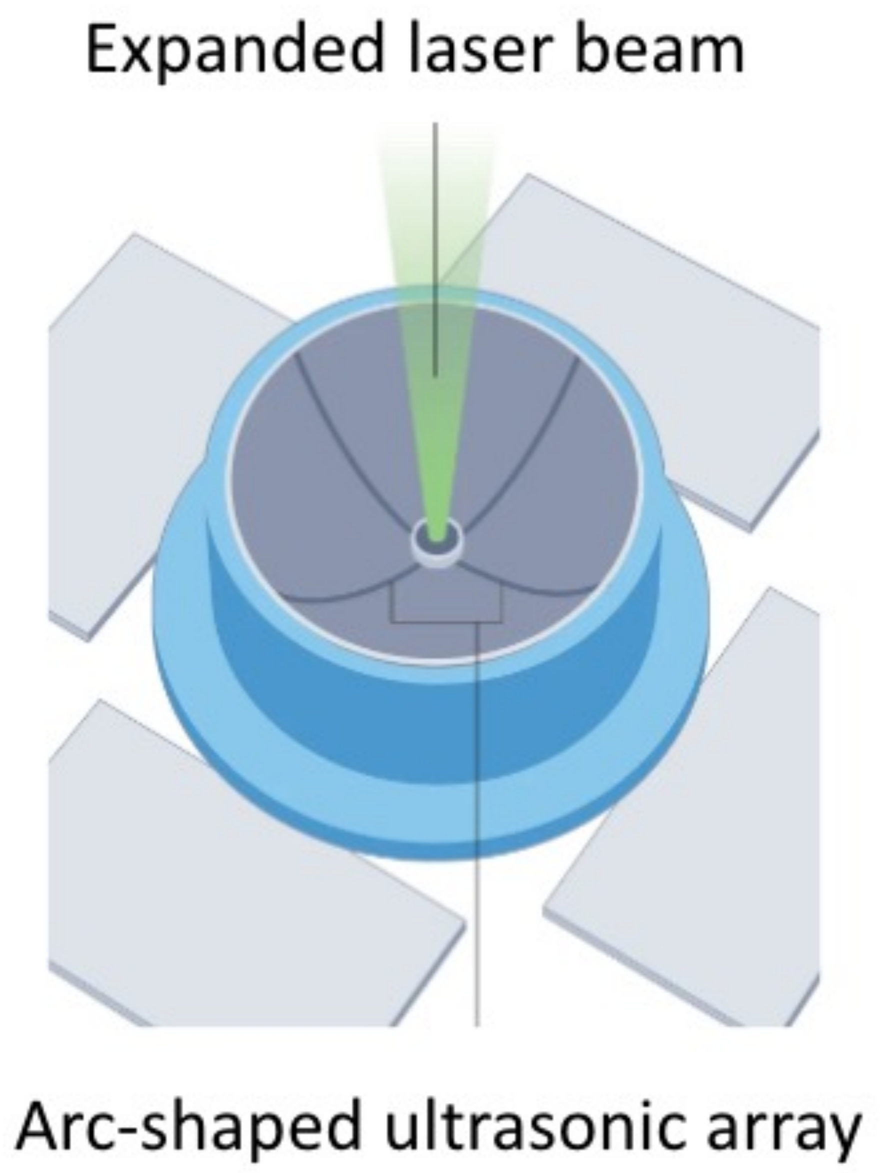
Same object, simulated RF and real RF



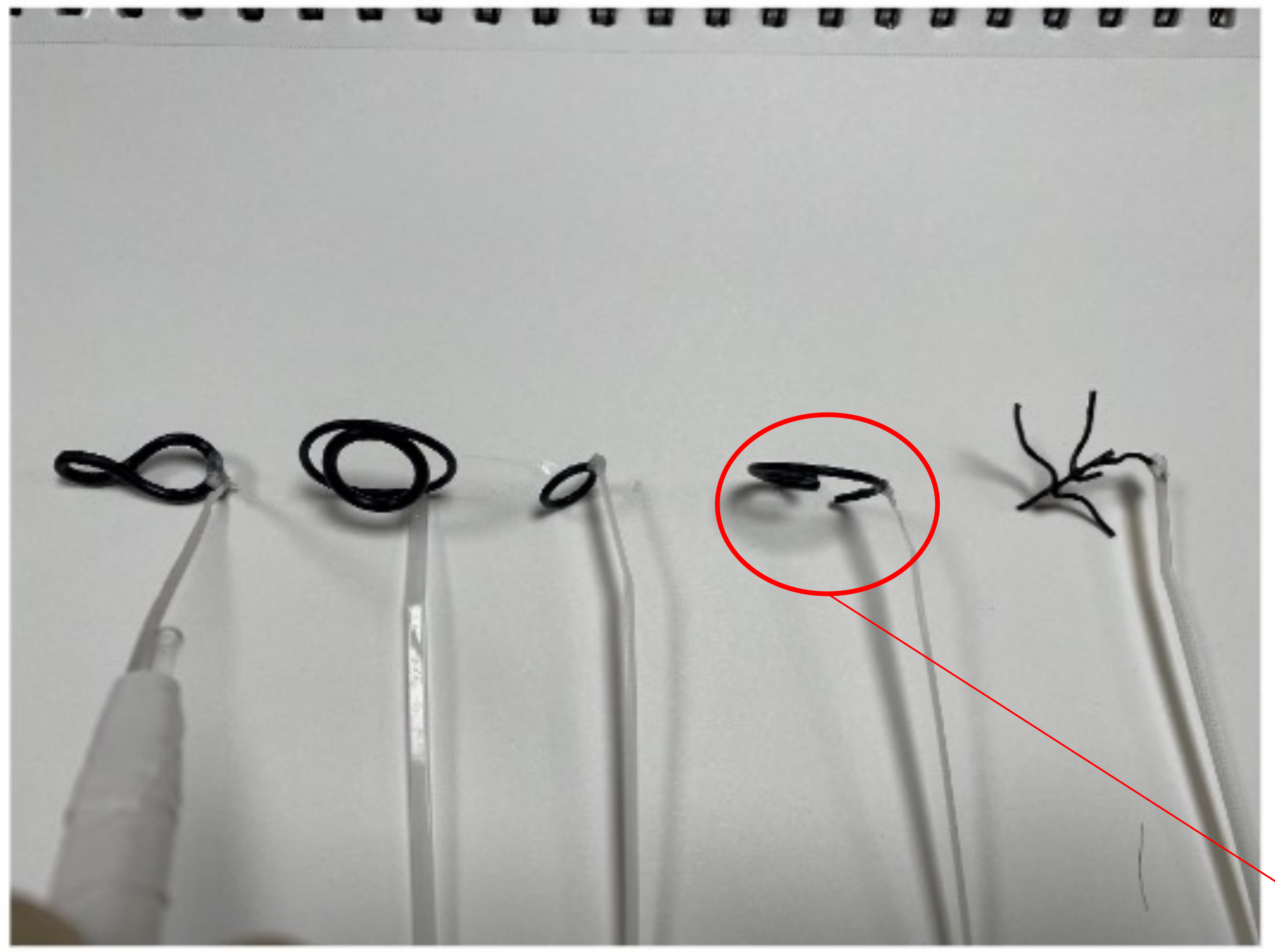


Experiments: Phantom Data

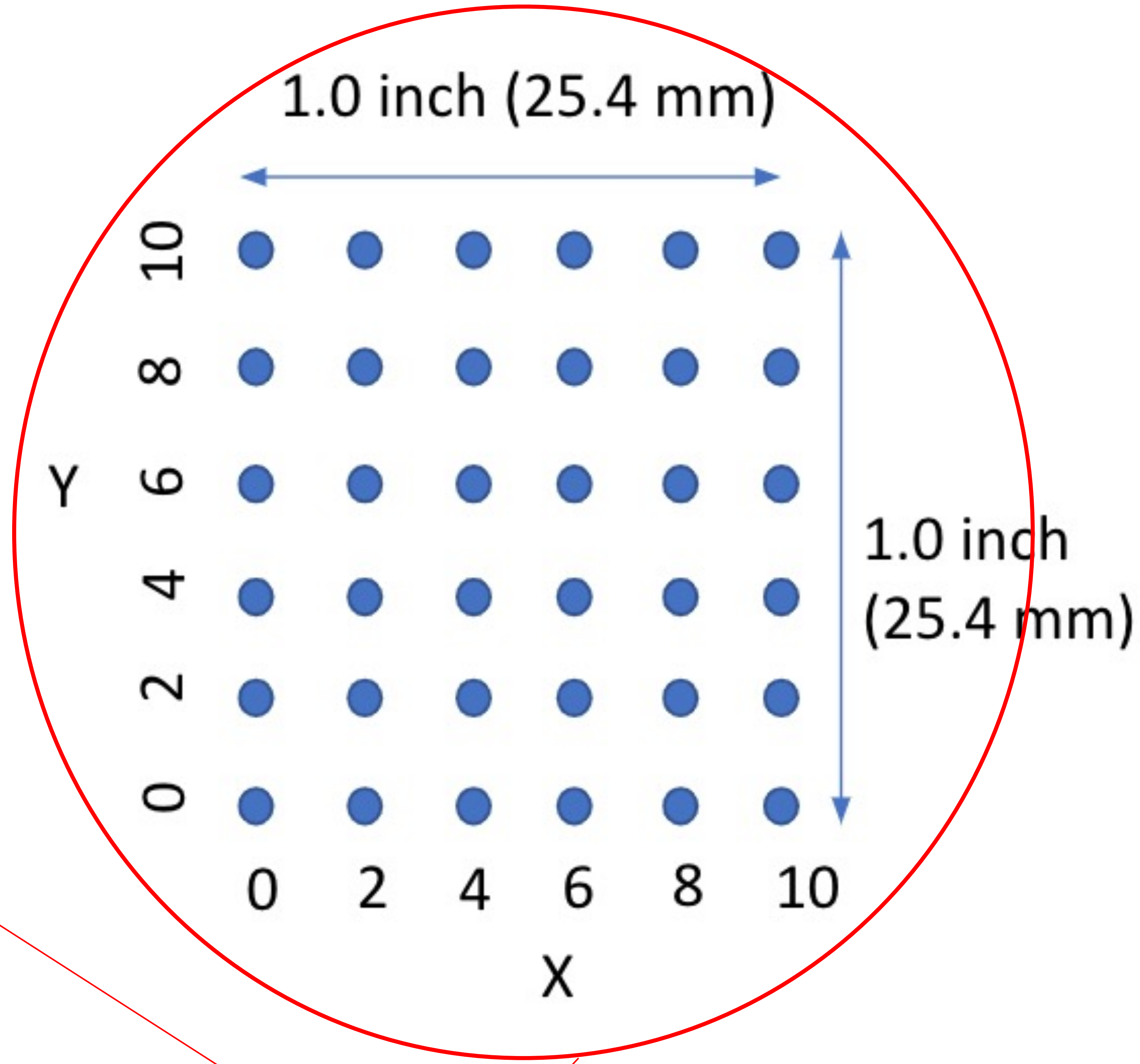
System



Phantom



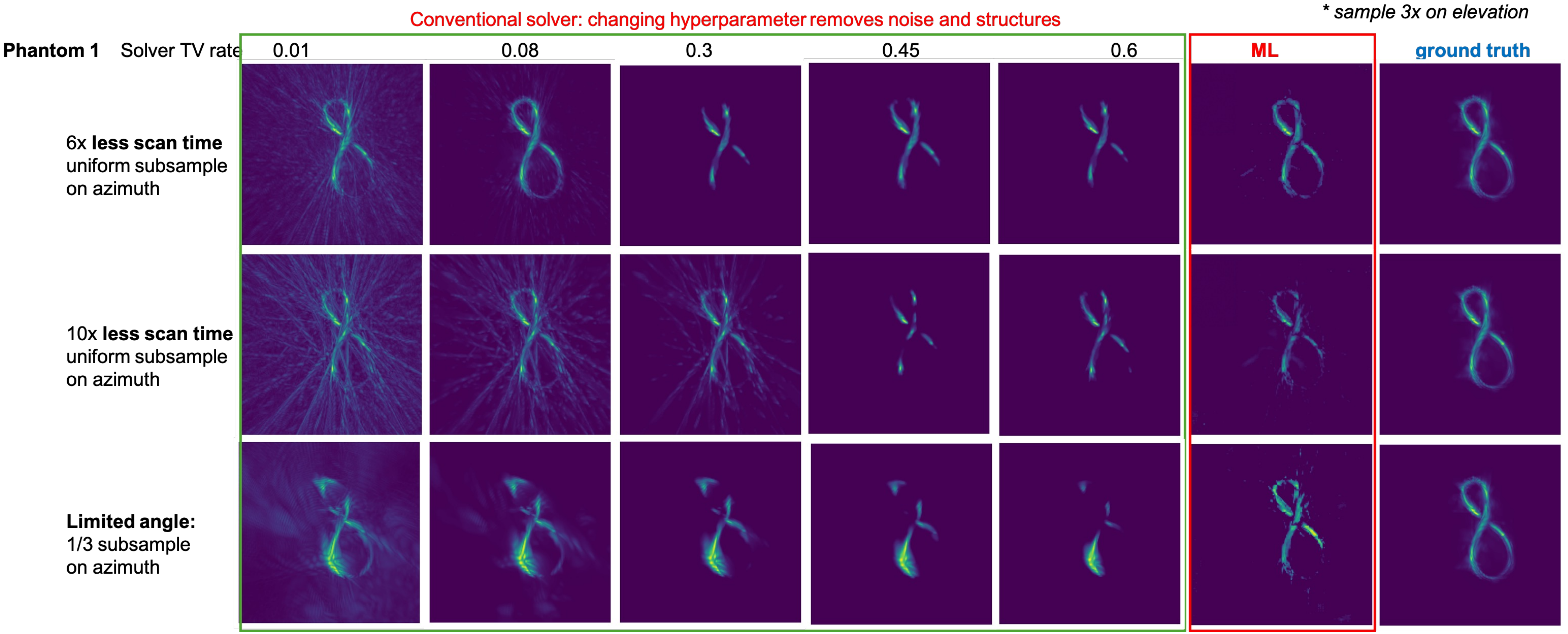
Point sources



Fine-tune

- Model first trained on 10k simulated samples
- Fine-tuned on one phantom combined with point source

Experiments: Phantom Data



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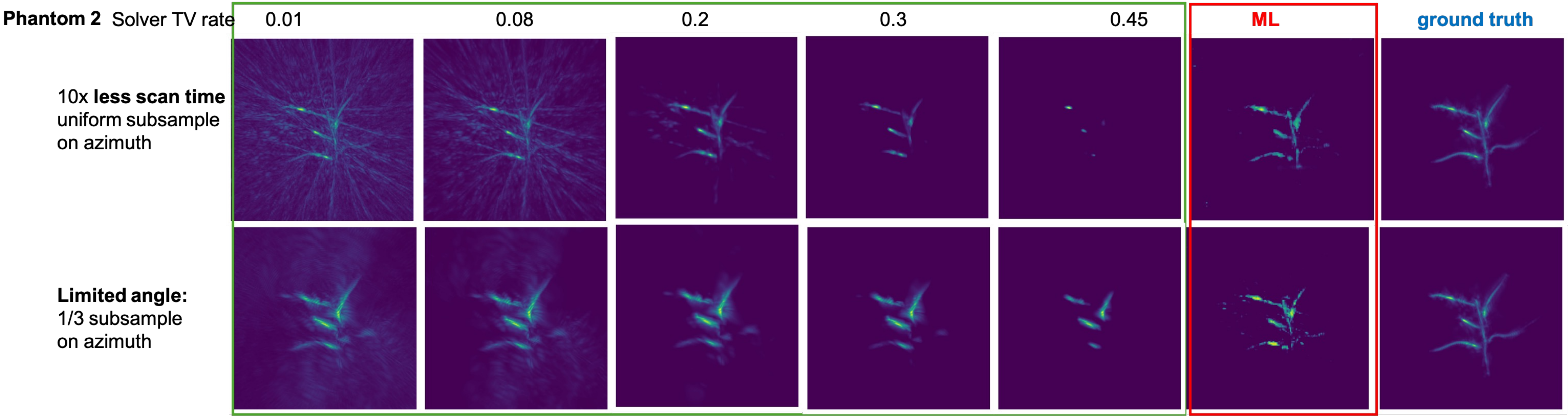

Summary

- Conventional solver needs to be tuned. Tuning removes noise and accurate phantom reconstruction simultaneously.
- ML has accurate reconstruction with less noise.
- Solvers needs to be tuned for individual phantom and individual setting.
- ML does not need individual-tuning.
- **Compression rate**: on real phantom, ML can reduce the full 10s scan time to 1s, or use 1/3 limited angle (120°). (Note the phantom structures are simpler.)